PRESSURE RESISTANT HULLS FOR SUBMARINA VESSELS
By: Rudolph N. J. Draaisma, 26 March, 1999

PRINCIPLE

The principle of this new technology, as described in the following, has much in common with the known story of the Dutch boy who saves his country from flooding by putting his thumb in a hole in the dike. In another version the same principle is known as the 'Hydraulic Paradox. The same principal applies also on sub marina vessels in which one could plug a small hole in the hull with a thumb, even at high external water pressures. The explanation is knowingly that the force working on the tiny area of that hole is negligible compared to the total force of the water pressure on the whole surface of the hull. For the same reason also a mechanical plug would not need to be designed as strong as the hull is on the whole.

The idea can be developed further by imagining that the whole hull consists of plugs that support each other side wise by wedging against each other. If these plugs were conical ones, there faces would press and seal against one another with greater force the larger the external water pressure gets. In this manner a hull could be composed of conical segments that are as thick or even thicker than what they are wide. Under such conditions it is evident that the strength of the hull is totally determined by the resistance against the surface stress between the faces of the segments, whereas the actual pressure load on the surface of the hull as such can be left out of consideration! As long as these segments do not crack, the hull is totally stiff and there is no risk for buckling, that normally sets the limits for the depth to which a submarine vessel can dive. With hulls of segment type it becomes absolutely possible to build manned submarine vessels that can reach the deepest bottoms of the oceans on Earth.

DESIGN & PROPERTIES

1) If the segments are geometrically symmetric they can be hexagonal blocks (6 conical faces) that compose a spherical hull. Deep diving submarine vessels generally have cylindrical hulls and then the segments can have a lengthy shape - beam segments with conical faces (see fig. 1). Though not apparent at first thought, it can be realized that beam segments become just as stiff as symmetrical blocks would, once they are assembled to form a cylinder. The side faces wedge against each other increasingly with the impact of the water pressure and the segments can therefore not bend inwards. No forces are generated in the length direction of the beams and with that no risk for buckling occur. For the same reason also spherical hulls or half-spherical sections can be composed of circular shaped beam segments with conical faces in radial direction. (see fig. 2), instead of the a.m. blocks. Mainly transversal press forces occur and therefore high resistance against pressure tensile stress is the decisive demand that is put upon the properties of the construction material.

Fig. 1
cylinder of beam segments

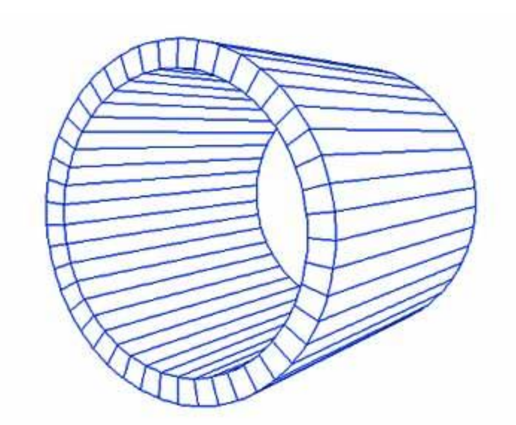
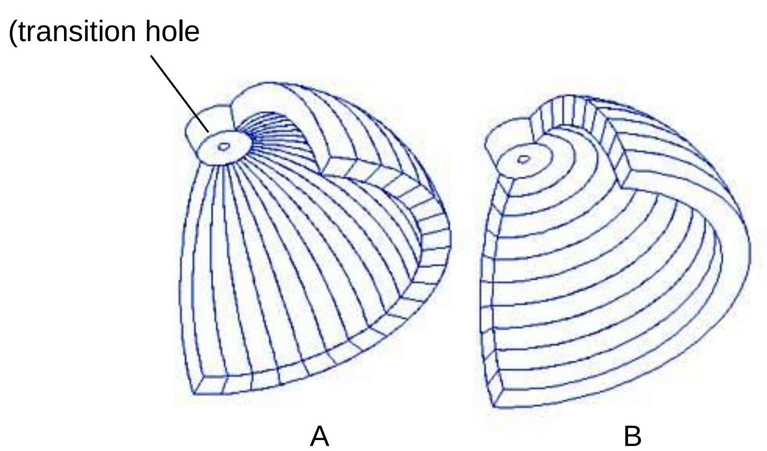


Fig. 2 length-section of spherical dome with:
A) radial beam segments
B) diametrical beam segments



Note: hull thicknesses not to scale.

2) When a cylindrical structure is closed with spherical domes, the water pressure on these domes will apply a longitudal force on the cylinder which at a first thought might lead to a buckling tendency of the beam segments. This is not quite so because the beams still cannot bend inwards and the impact of the water pressure acts against bending outwards (the free buckling length acc. to Eulers theorem is sort of zero). Furthermore the longitudal tensile stress cannot become larger than the transversal tensile stress on the beam faces. A calculation shows actually that the ratio of these stresses is constant and equals two, meaning that the surface tensile stress in the joints between the domes and the cylinder body is only half of the tensile stress between the faces of the beam segments. The only event that could be imagined would be a plastic deformation of the construction material, for which reasons metals and other materials with elastic properties are not suitable for large diving depths. For the same reason it would not be advisable to armor the segments with steel rods in length direction. If these rods were to absorb the a.m. buckling forces they would tend to bend out by their own elasticity and weaken the segments instead of reinforcing them! The resistance against pressure tensile stress of the construction material of the segments (concrete) could however be largely improved with (glass)fiber reinforcements that minimizes the brittleness.

3) If the conicity of the side faces is less than 5 angular degrees, the height (thickness) of the segments at given water pressure is proportional with the diameter of the hull and inverse proportional with the allowed surface tensile stress. Calculations show that within certain restraints, the contact force between the faces is independent from the number of segments. This is due to the circumstance that varying the number of segments also varies their conicity and the water pressure load upon them, which takes each another out.
Because the number of segments is to be set as such that the surface tensile stress between them becomes the dominating load condition on the material, also the height (thickness) of the segments becomes independent from the number of segments within the restraint.

If a lower number of segments is set, such that other load stresses become of consideration for the thickness of the segments, one has passes beyond the intention of the segment method for which reason this alternative can be discarded. Within the a.m. restrictions it can be said in general that the number of segments should be set such that their cross-section not becomes wider than what it is high; preferably the height should be larger than the width. The characteristic design properties are furthermore that for given design parameters the thickness of the segments varies proportionally and consequently their mass (weight) to the square of the diameter of the hull. In addition, for a given diameter the thickness and the mass (weight) vary inverse proportional with the allowed surface tensile stress.

The remarkable thing is that, contrary to conventional construction methods, the hull (segment) thickness is largely independent from the shape of the hull. The explanation is that the actual surface load on the hull does not constitute the dimensioning load on the segments of which it is composed; it is only the prevailing contact forces between the segments that is dimensioning for the hull thickness. These forces are generated by the water pressure on the size of the exposed surface of each segment and not by the shape of this area.
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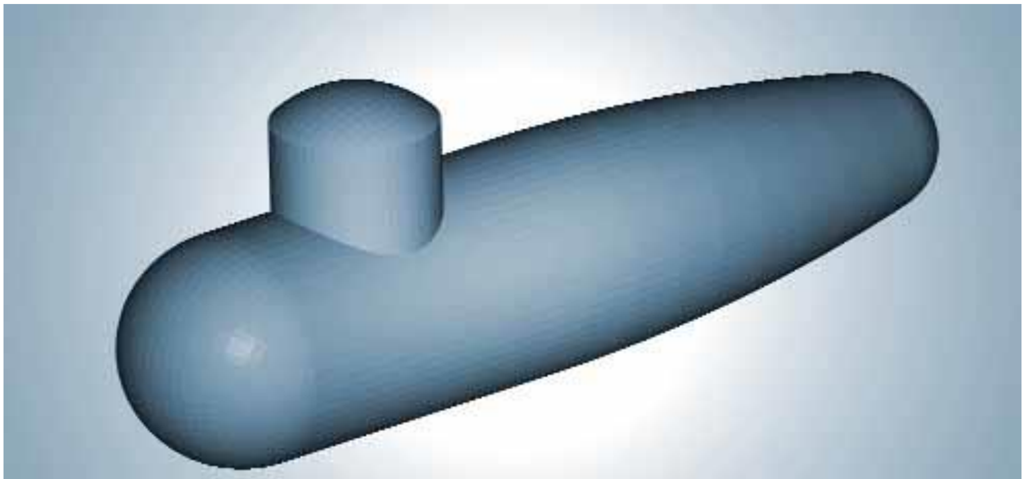
The diving depth of a segmented hull does not depend on its shape !



Segment hull, composed of spherical, straight- and arc-cylindrical elements

In consequence with the above it can be concluded that the segment method is a less suitable construction method for hulls with a diameter less than one (3' 4") meter and should definitely be replaced by conventional methods for diameters below half a meter (1' 8"). The reason is that such hulls would become a rather small segment thickness which then would imply a too small width in order to keep the cross-section at least quadratic. The manufacturing of segments with such small cross-section would make the method unsuitable for practical reasons. Decreasing of the number of segments in order to make them wider would also imply a larger thickness which makes the hull more heavy than what it otherwise need to be, plus that the intention of the method is passed by.

4) The amazing with this segment construction method is that a segmented hull gets stiffer, i.e. becomes more resistant against deformations, the greater the external water pressure gets. With conventional design methods the inverse is valid.
A segmented hull can only be crushed, it cannot be deformed !

This is not only an advantage. The disadvantage is the need to apply the method in such a manner that an underwater vessel also is stiff enough in surface position and not in the least when it is docked dry.
Under such conditions the hull becomes subjected to longitudinal bending stresses by virtue of its own weight. This causes stretching tensile in the concrete that can cause cracks and even ruptures. I surface water position such tensile can be minimized through a shaping by which the mass of the vessel varies accordingly with the displacement along the water line. A basically cylindrical hull can have a varying diameter by the segment beams having a certain curvature. These circumstances also have consequences how and where heavy inventories, such as engines, are placed in the hull in respect to the distribution of loads. When a vessel with segmented hull is docked dry, it would not be appropriate to support it at a few locations only but should be supported over its full length instead. Naturally this applies mostly on large vessels and depending on how lengthy they are. A great plus in this respect is that with an appropriate manufacturing method (see below) an eventual occurrence of cracks do not have to have an impact on the diving depth ability of the vessel if the edges, even at rupture, cannot move or glide inwards (the faces of ruptured parts are still wedging against adjacent faces).

CONSTRUCTION METHODS

1) In the following I focus on larger, preferably manned underwater vessels and that the material of the segment beams is fiber reinforced concrete. Although other types of materials can be considered, they might and likely will demand other manufacturing methods then those I describe here, for which reason I limit myself to concrete. In the above the behavior of segmented hulls has been focused on the condition of being pressurized under water, but they must also keep together under surfaced conditions. If we just think in terms of loose segments that have to be mounted and kept together, the whole thing looks rather problematic.

To start with, there would be a need for a inner or outer structure that keeps the shape during manufacturing. The segments have then to be attached to this structure in a suitable manner which also should be strong enough to keep the segments pressed against each other even against gravity. This will surely become an excessively strong structure that on top of it is not needed in diving position. After all, the segments do then constitute a self-supporting hull under impact of the water pressure. It is astonishing any way that we discuss a design method here for submarine vessels that basically are weaker under surfaced than under submerged conditions.

2) Secondly the requirements on precision and smoothness of the segment faces become very high. Even though, it would not be suitable to place the segments directly against each other, as any unsmoothness or impurities, such as particles or sand corns that get in between, would or could initiate the formation of cracks and ruptures under high contact forces. There ought be an equalizing layer between the faces of the segments that also must stand the contact stresses under submerged conditions - maybe straps of fiberglass ? If these equalizing layers are of elastic materials such as thin sheets of aluminum for example, the segments under stress might plasticize this material and glide inwards however little - what are the allowable criteria and how to solve the attachment of the segments in or on the supporting structure under such conditions during manufacture?

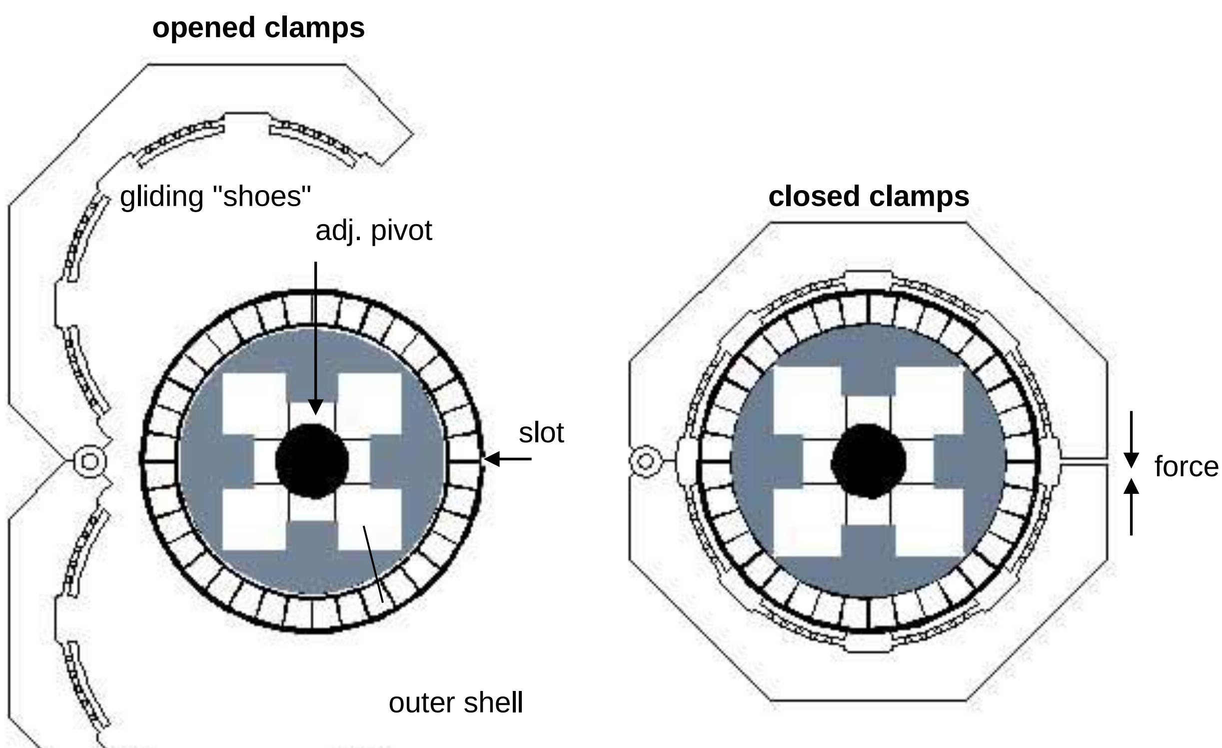
For the a.m. reasons it would be better to apply an inner and an outer shell in which longitudinal partitions are placed radially that create cavities in which concrete is cast. The big advantage of this method is of course that requirements on precision and smoothness for pre-fabricated segments can be ignored all together as well as that a perfect fitting of the segments is obtained. Instead we get requirements on a casting procedure that assures a ho~~mogeneous~~ and appropriate packed filling. The longitudal partitions do no longer have an 'equalizing' function as there is nothing to equalize. The partitions are now just partitions to physically separate the segments and could theoretically have a thickness zero and be removed aver curing of the fillings. As they unavoidably will have a certain thickness and remain in the structure, I propose to use the hardest and most inelastic material that I can think of, being (armored) glass. It is said that glass is a 'liquid' and I don't know if and what consequences this may have for the application, not in the least in terms of aging (it is observed that window glass in very old buildings have become thicker at the underside) Perhaps the glass also can be fiber reinforced or other materials must be applied

1) CYLINDRICAL SHELLS (fig.3)

The inner shell can be of relatively thin plate material if it is placed over an adjustable pivot that keeps the shape during casting. The outer shell should be somewhat thicker and have a n open slot over it full length, the size of which depends on the diameter. This slot can be sealed with a plastic strip during casting and be removed later. After completed casting the outer shell is then catched between clamps around the shell so that the still wet concrete is squeezed inwards and guarantee a homogeneous packing and perfect distribution of the filling in all of the cavities. Of course there should be a minimal clearance between the partitions and the shell that allows the inward displacement. After the concrete has cured completely the outer shell is pressed further around the body as to close the slot after which the latter can be welded to join. Through this the following features are obtained:

a) The segments are pressed against each other and give the shell body a "dry" stiffness.
b) The outer shell is stretched elastic and obtain a pre-tensile that assures that the segments stick together even within a certain temperature range and under other conditions that otherwise could increase or deform the diameter or shape of the shell in either direction.

Fig. 3



Obs: the gliding shoes and the according parts of the adjustable pivot can be replaced for smaller diameters

The egg of Columbus:

1) Through this procedure, the as such formed body obtains totally astonishing properties; it does not have an additional load impact by the water pressure down to a depth that is according to the force applied to close the slot in the outer shell during manufacturing! At this depth the tensile stress in the welded joint has decreased to zero; the plate material of the outer shell is now pressed around the segments by the water pressure, instead of being pulled around them by forces working in and from the joint.

One would be inclined to assume otherwise that the load on the segments is the sum of the water pressure force and the stretching force resulting from manufacture but in fact it is true that the water pressure force applies the same impact that the clamps did in the casting mould. One could also see it in another way: as the water pressure increases, the tensile stress in the welded joint decreases and with that the force by which the joint pulls the plate of the outer shell around the segments of the body decreases. Or, in other words: one could cut the plate in underwater position and the plate material would still be pressed against the segments of the body with unchanged force (provided no water enters between the plate and the body).

The tensile stress in the plate material remains unchanged the same, only the cause of it has become another one. This implies that the hull doesn't "feel" that it is submerged onto a certain depth. Only at larger depths the load on the segment body becomes larger than it was from manufacture. From this follows that the "free" diving depth can be increased by applying high strength steel or ditto aluminum with a larger thickness that allows higher tensile stresses during manufacturing of the hull.

2) The astonishments do not end here; at further increasing water pressures the stretching stress in the plate material of the outer shell will gradually decrease! The force of the water pressing against the shell tends to sort of "smear" out the plate material over the body through which the thickness of the plate material would become less if the material had somewhere to go (for example in an overlap that increases). Because the material has nowhere to go its thickness cannot decrease and instead the stretching stresses in the material decreases to finally become zero and change into pressing (buckling) stresses at even larger depths. Meanwhile, this has hardly any impact on the strength of the hull because the zero tensile stress only occurs if the segments of the body haven't accommodated their positions even the least. Should that happen to an however little extend, the plate material would deform immediately and accordingly. On the other hand, the tensile in the plate material can be measured with sensors and give the crew of the vessel reassuring or alarming indications about the condition of the hull under load.

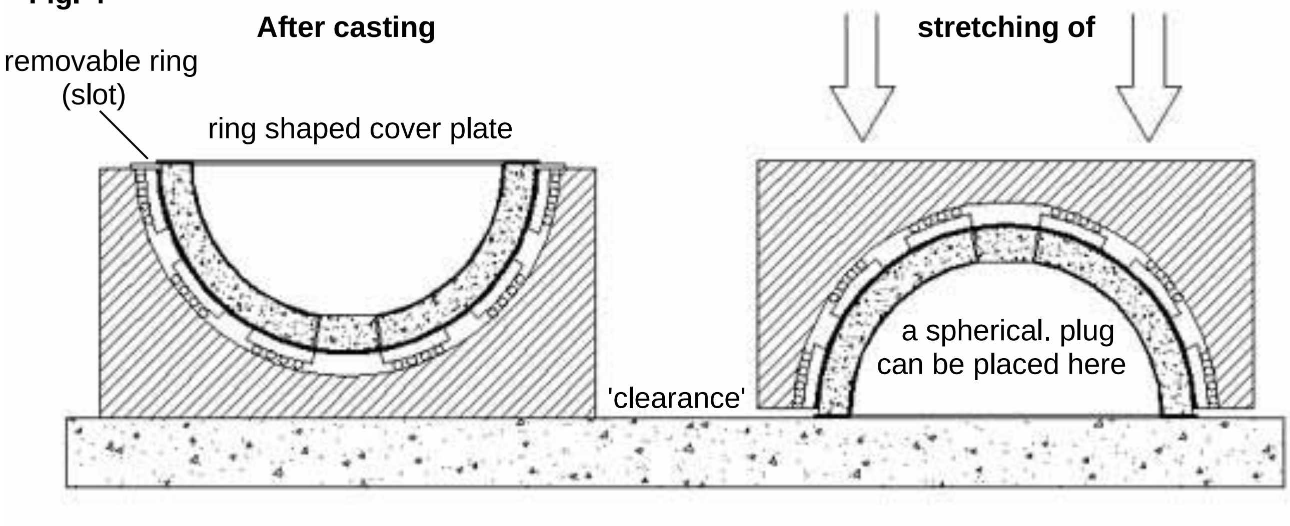
SPHERICAL HULLS AND DOMES (fig. 4)

It is obviously much more complicated to cast and stretch spherical shapes than cylindrical ones. Instead pre-fabricated conical, diametrical segments (acc. fig.2) with varying diameters could be applied to be placed upon each other in a half-spherical shell. A completely spherical hull is than obtained by assembling two half-spheres together. Equalizing strips and layers have to be applied between the segments and the outer plate shell as mentioned before. A further big disadvantage of such a method is that all segments are different from each other and that for different hull diameters other sets of segments are required. Furthermore, these diametrical segments must be positioned radially because the shape-stiffness of closed ring segments would otherwise inhibit contact forces to arise between the faces of the segments. On the other hand, by partitioning the ring segments we basically come back to a design with radial segments (oops - we may discard diametrical segments all together but for a better understanding, it is good to realize this.) ! Because of these manufacturing difficulties, I rather describe a cast method for radial segments.

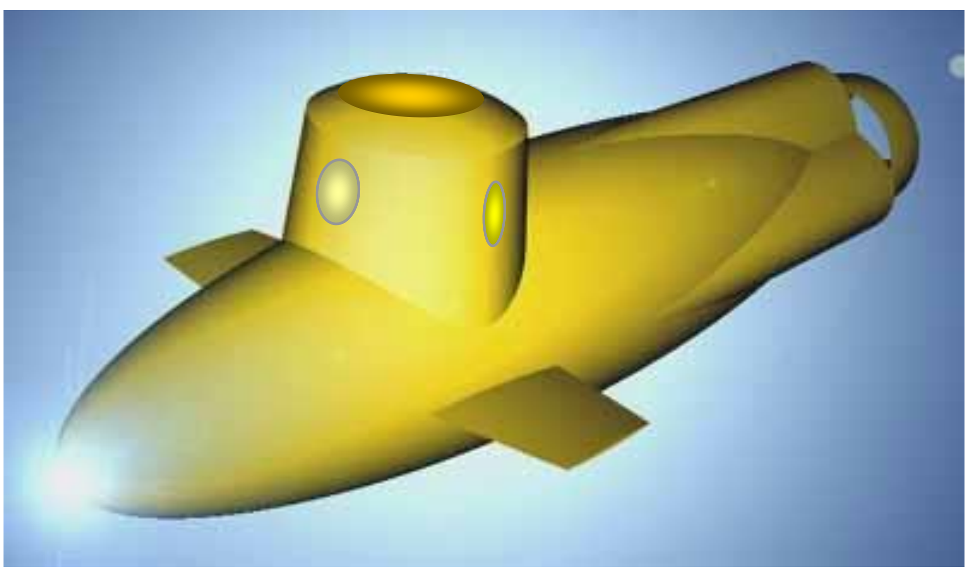
The method is basically the same as for cylindrical hulls; we have an outer and an inner shell in between which arc shaped partitions are placed radially. These partitions rather should not meet at the pole of the sphere and there form all too small cavities that complicate casting. It is better to have a conical plug as a pole that then also is suitable to have a transition hole, for example to accommodate a propeller shaft. Casting is done in a fixture with the diameter of the half-sphere turned upwards. The inner shell is lowered between the transitions and keeps a certain clearance from them that allows to lower it a little more after completed casting. When that is done a ring shaped cover plate is pushed on the still wet concrete and through all this the fillings are packed and uniformly distributed in the cavities.
The outer shell has a removable ring at its upper, largest diameter and has the same function as the slot in the outer shell of a cylindrical hull.

The thickness of this ring is on the contrary much less than the slot would be wide. When the concrete has cured completely this ring is removed, the ring shaped cover plate is welded on to the inner shell, after which the whole fixture is turned upside down.
Now the outer shell can be pressed down to close the spacing (slot) that remains after the a.m. ring was removed. In the process the material of the outer shell will be stretched over the segments and press the latter against each other in a likewise manner as was done with cylindrical hulls.

Fig. 4



As the material is stretched 2-dimensionally it will likely become necessary to do it in warm condition so that the shell will shrink around the segments as it cools down. Before this happens, the joint between the cover plate and the outer shell is welded, whilst the outer shell is still pressed down towards the joint. I am aware that this is a very rough description and that many details remain to be solved, but this has to be done by experts who can do that better than I.



Segmented hull: elliptical longitudinally, circular transversally

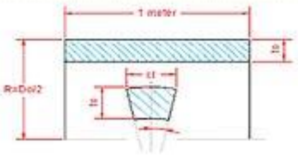
I have designed a calculation program in MathCad, that allows you to dimension different hull shapes for various conditions - see a print-out below. You can purchase this program from me (for 50 USD only) and/ or give me an assignment to calculate a design of your preference. You can also hire me as a consulting engineer, to develop this concept further into a commercial design.

Send your inquiries to: rudolph@draaisma.net
website: http://www.draaisma.net/alternative_engineering/index.html

CYLINDRICAL SHELL (base calculation)

Enter parameters: (example calculation)

max. diving depth	$d := 11000$	meter	number of segments	$n := 100$	
Allowed material stress	$\sigma := 10$	N / mm²	Shell outside diameter	$Do := 4000$	mm
Modulus of elasticity	$E := 2000$	N / mm²	material density	$\rho := 2400$	kg / m³
allowed surface tensile stress	$S := 1000$	N / mm²	material	fiber reinf. concrete	



$\alpha := \frac{\pi}{n}$ cone angle

$P := \frac{d}{100}$ pressure in N / mm²

$R := \frac{Do}{2}$ Shell radius

$cl := 2 \cdot R \cdot \sin(\alpha)$ segm. cord length

for bending: $tb := R \cdot \sin(\alpha) \cdot \sqrt{\frac{P}{\sigma}}$ for buckling (Euler): $tk := R \cdot \sqrt[3]{\frac{48 \cdot P \cdot (\sin(\alpha))^2}{E \cdot \pi^2 \cdot \cos(\alpha)}}$ $t := \mathrm{if}(tb > tk, tb, tk)$ 't' is largest of tb, tk

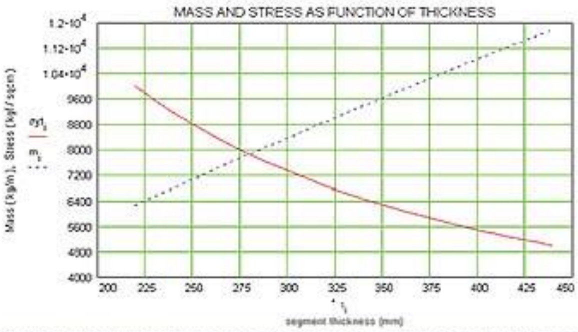
$ts := \frac{R \cdot P}{S}$ for surface tensile stress $to := \mathrm{if}(ts > t, ts, t)$ 'to' is largest of t, ts $Di := Do - 2 \cdot to$

$Dpl := \pi \cdot R^2 \cdot 1 \cdot 10^{-6}$ $m := \pi \cdot \left[R^2 - (R - to)^2\right] \cdot \rho \cdot 1 \cdot 10^{-6}$ hull mass (weight) per meter length of hull

HULL DIMENSIONS: inside diameter	Di = 3560	mm	**segment height**	to = 220	mm
mass (weight)	m = 6270	kg/m	**cord length**	cl = 126	mm
Displacement	Dpl = 12.6	m³/m			

ANALYSIS :

$i := 0, 1 .. 20$ $k_i := 1 + \frac{i}{20}$ $t_i := k_i \cdot to$ $m_i := \pi \cdot \left[R^2 - (R - t_i)^2\right] \cdot \rho \cdot 1 \cdot 10^{-6}$ $\sigma yt_i := \frac{R \cdot P}{t_i} \cdot 10$



From the curves above we see that for given diameter, number of segments and diving depth, the mass (weight) of the shell is proportional and the allowed tensile surface stress inversed proportional with the thickness (height) of the segments.

In case the curvature of the graph for σyt is confusing, one should realize that the allowed stress becomes infinite at zero thickness and the graph can therefore not be a straight line.)

REMARK: With the calculations in this worksheet I have made the observation that there is a distinct ratio between the mass and the displacement (mass of water) of a segmented shell, regardless its shape and size, with the diving depth (water pressure) and the number of segments (n) as parameters:

Depth (meter)	Mass	n=100 Displ	n = 50 Displ
11000	1	2	1
5500	1	3	1.5
2750	1	4	2
1375	1	6	3
700	1	8	4

This means that the number of segments should be kept as large as possible within manufacturing and/or economical restraints.

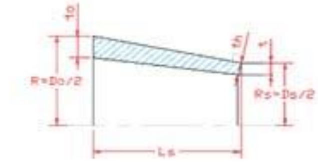
This also shows that the calculations reflect the correct characteristics of the segmented shell, as they logically are emphasized the more a shell is segmented.

SHELLS WITH VARYING DIAMETERS OF CIRCULAR CROSS-SECTION
(connected to the cylindrical shell as calculated above)

Shells with varying diameters of circular cross-sections can have length-sections that are conical, elliptical or arc shaped. If the radius of the arc is the same as that of the circular cross-section, the shell becomes a spherical one. The beam segments of such hulls become less wide at lower cross-section diameters and hence their thickness can become lesser, according to what would be valid for a cylindrical shell with the same diameter.
From the above calculations for cylindrical hulls we learn that the segment thickness (height) is proportional with the shell radius. This property is used in the following calculations. The thickness of the segments at each cross-sectional radius is calculated to be the length of the intersection along the normal of the curvature with a segmented cylinder at that location.
(this procedures gives as a result, that the hull of spherical shells obtain a constant thickness, equal to that of the largest cross-section.)

CONICAL SHELL SECTIONS :

Enter smallest outside diameter $Ds := 2000$ mm
length of section $Ls := 3317$ mm



$Rs := \frac{Ds}{2}$ $t := \frac{Rs}{R} \cdot to$

$cs\phi := \frac{Ls}{\sqrt{Ls^2 + (R - Rs)^2}}$

$Dpl := \frac{\pi \cdot Ls}{3} \cdot \left[(R)^2 + (Rs)^2 + (R \cdot Rs)\right] \cdot 10^{-9}$ $th := \frac{t}{cs\phi}$

$v := \frac{\pi \cdot Ls}{3} \cdot \left[(R)^2 + (Rs)^2 + (R) \cdot (Rs) - (R - to)^2 - (Rs - th)^2 - (R - to) \cdot (Rs - th)\right]$ $m := v \cdot \rho \cdot 1 \cdot 10^{-9}$

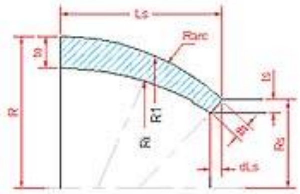
Largest diameter of shell	Do = 4000	mm	**Length of section**	Ls = 3317	mm
Smallest diameter of shell	Ds = 2000	mm	**Mass (weight) of section**	m = 12277	kg
Thickness at smallest diameter	th = 115	mm	**Displacement of section**	Dpl = 24.3	m³

ARC SHAPED SHELL SECTIONS

(spherical shape for Rarc = R = 2000 mm)

Enter: Radius of outer arc $Rarc := 6000$ mm
Smallest diameter of section $Ds := 2000$ mm

$Rs := \frac{Ds}{2}$



$Ls := \sqrt{Rarc^2 - (Rs + Rarc - R)^2}$

Length of section Ls = 3317 mm

$tg\phi s := \frac{Ls}{(Rs + Rarc - R)}$

$dLs := \frac{to \cdot Rs}{R} \cdot tg\phi s$

$e := 50$ $z := 0, 1 .. e$ $u := Rarc - Ls$ $u := \mathrm{if}(u \ge 500, 500, u)$ $L := Ls + u$ $L_z := L \cdot \frac{z}{e}$

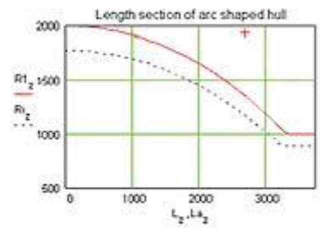
$R1_z := \sqrt{Rarc^2 - (L_z)^2} - Rarc + R$ $R1_z := \mathrm{if}(R1_z < Rs, Rs, R1_z)$ $t_z := to \cdot \frac{R1_z}{R}$ $ts := to \cdot \frac{Rs}{R}$

$tg\phi_z := \frac{L_z}{R1_z + Rarc - R}$ $La_z := L_z + t_z \cdot tg\phi_z$ $Ri_z := R1_z - t_z$ $cs\phi := \frac{Rarc - R + Rs}{Rarc}$

$th := \frac{ts}{cs\phi}$ $vi := \pi \cdot \int_0^{Ls - dLs} \left[\left[\sqrt{Rarc^2 - (Ls)^2} - Rarc + R\right] - to \cdot \left[\frac{\sqrt{Rarc^2 - (Ls)^2} - Rarc + R}{R}\right]\right]^2 dLs$

$v1 := \pi \cdot \int_0^{Ls} \left[\sqrt{Rarc^2 - (Ls)^2} - Rarc + R\right]^2 dLs$ $v := v1 - vi$ $m := v \cdot \rho \cdot 10^{-9}$ $Dpl := v1 \cdot 10^{-9}$

Largest diameter of shell	Do = 4000	mm
Smallest diameter of shell	Ds = 2000	mm
radial thickn. at smallest diameter	th = 132	mm
Length of section	Ls = 3317	mm
Mass (weight) of section	m = 15556	kg
Displacement of section	Dpl = 30.3	m³

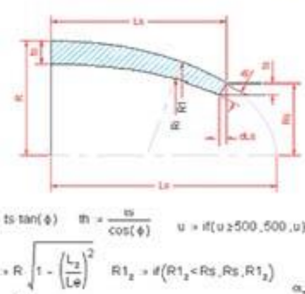


ELLIPSOIDAL SHELL SECTIONS

(spherical shape for Le = R = 2000 mm)

Enter: Half of largest axis of ellipsoid $Le := 3830$ mm
Smallest cross-section diameter of section $Ds := 2000$ mm
calculated length of section Ls = 3317 mm

$Ls := Le \cdot \sqrt{1 - \left(\frac{Ds}{2 \cdot R}\right)^2}$



$Rs := \frac{Ds}{2}$ $c := \sqrt{Le^2 - R^2} + 1$

$u := Le - Ls$ $\alpha := \mathrm{atan}\left(\frac{Rs}{Ls - c}\right)$

$\beta := \mathrm{atan}\left(\frac{Rs}{Ls + c}\right)$ $ts := to \cdot \frac{Rs}{R}$

$\gamma := \mathrm{if}(\alpha \ge 0, \alpha - \beta, \pi - |\alpha| - \beta)$

$\phi := \mathrm{if}\left[Ls > c, \frac{\gamma}{2} + \frac{\pi}{2} - \alpha, \frac{\gamma}{2} - \left(\frac{\pi}{2} - |\alpha|\right)\right]$

$dLs := ts \cdot \tan(\phi)$ $th := \frac{ts}{\cos(\phi)}$ $u := \mathrm{if}(u \ge 500, 500, u)$ $L := Ls + u$ $L_z := L \cdot \frac{z}{e}$

$R1_z := R \cdot \sqrt{1 - \left(\frac{L_z}{Le}\right)^2}$ $R1_z := \mathrm{if}(R1_z < Rs, Rs, R1_z)$ $\alpha_z := \mathrm{atan}\left[\frac{R1_z}{(L_z - c)}\right]$ $\beta_z := \mathrm{atan}\left(\frac{R1_z}{L_z + c}\right)$

$t_z := to \cdot \frac{R1_z}{R}$ $Ri_z := R1_z - t_z$ $\gamma_z := \mathrm{if}(\alpha_z \ge 0, \alpha_z - \beta_z, \pi - |\alpha_z| - \beta_z)$

$\phi_z := \mathrm{if}\left[Ls > c, \frac{\gamma_z}{2} + \frac{\pi}{2} - \alpha_z, \frac{\gamma_z}{2} - \left(\frac{\pi}{2} - |\alpha_z|\right)\right]$

$v1 := \pi \cdot \int_0^{Ls} R^2 \left[1 - \left(\frac{Ls}{Le}\right)^2\right] dLs$ $vi := \pi \cdot \int_0^{Ls} \left[R \cdot \sqrt{1 - \left(\frac{Ls}{Le}\right)^2} - to \cdot \frac{R \cdot \sqrt{1 - \left(\frac{Ls}{Le}\right)^2}}{R}\right]^2 dLs$

$La_z := L_z - t_z \cdot \tan(\phi_z)$

$Dpl := v1 \cdot 10^{-9}$ $v := v1 - vi$ $m := v \cdot \rho \cdot 10^{-9}$

Largest diameter of shell	Do = 4000	mm
Smallest diameter of shell	Ds = 2000	mm
radial thickn. at smallest diameter	th = 148	mm
Length of section	Ls = 3317	mm
Mass (weight) of section	m = 15598	kg
Displacement of section	Dpl = 31.3	m³